A Decomposition Theorem for Dynamic Flows

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Abstract

The famous edge flow decomposition theorem of Gallai (1958) states that any static edge s,d-flow in a directed graph can be decomposed into a linear combination of incidence vectors of paths and cycles. In this paper, we study the decomposition problem for the setting of dynamic edge s,d-flows assuming a quite general dynamic flow propagation model. We prove the following decomposition theorem: For any dynamic edge s,d-flow with finite support, there exists a decomposition into a linear combination of s,d-walk inflows and circulations, i.e. edge flows that circulate along cycles with zero transit time. We show that a variant of the classical algorithmic approach of iteratively subtracting walk inflows from the current dynamic edge flow converges to a dynamic circulation. The algorithm terminates in finite time, if there is a lower bound on the minimum edge travel times. We further characterize those dynamic edge flows which can be decomposed purely into linear combinations of s,d-walk inflows.
The proofs rely on the new concept of autonomous network loadings which describe how particles of a different walk flow would hypothetically propagate throughout the network under the fixed travel times induced by the given edge flow. We show several technical properties of this type of network loading and as a byproduct we also derive some general results on dynamic flows which could be of interest outside the context of this paper as well. Joint work with Lukas Graf and Julian Schwarz.

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