
A characterization of unimodular hypergraphs with disjoint hyperedges

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Abstract

Grossman et al. show that the subdeterminants of the incidence matrix of a graph can be characterized using the graph's odd cycle packing number. In particular, a graph's incidence matrix is totally unimodular if and only if the graph is bipartite. We extend the characterization of total unimodularity to disjoint hypergraphs, i.e., hypergraphs whose hyperedges of size at least four are pairwise disjoint. Disjoint hypergraphs interpolate between graphs and hypergraphs, which correspond to arbitrary $\{0, 1\}$ -matrices. We prove that total unimodularity for disjoint hypergraphs is equivalent to forbidding both odd cycles and a structure we call an "odd tree house". Our result extends to disjoint directed hypergraphs, i.e., those whose incidence matrices allow for $\{0, \pm 1\}$ -entries. As a corollary, we resolve a conjecture on almost totally unimodular matrices, formulated by Padberg (1988) and Cornuéjols & Zuluaga (2000), in the special case where columns with at least four non-zero elements have pairwise disjoint supports.

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